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ocally identified many low-dimensional deterministic phenomena in population data. These phenomena include equilibria, cycles, transitions between dynamic regimes (bifurcations), multiple attractors, resonance, basins of attraction, saddle influences, stable and unstable manifolds, transient phenomena, and even chaos. Robust qualitative and quantitative predictions have become possible for several laboratory systems; see, for example, [**1**]–[**5**], [**8**], and [**10**]–[**12**].

A major goal of laboratory studies, of course, is to gain clear insights that might be applied to fluctuations in field populations. Despite the very real difficulties of developing quantitatively accurate models for field systems, many researchers are optimistic that we are gaining the necessary conceptual tools and insights. If some of the recent successes in the laboratory can be extended to the field, unprecedented advances in field ecology may lie just around the corner.

So what does this have to do with textbooks for mathematical modeling? In this exciting climate of accelerating change, students of biology in general and ecology in particular should be trained in the mathematical methods just as physics majors are. Interdisciplinary courses on mathematical models in biology are springing up at many university campuses. These classes are important to the future of the discipline of ecology. Not all the students thus trained will go on to do mathematical modeling in their careers; but hopefully they will have lost any prejudice they might have harbored against the method of abstraction and will point their own students to the importance of mathematical training. In other words, classes in mathematical modeling can help change the academic culture of biology and ecology departments.

I have had the pleasure of teaching such courses at the College of William and Mary and Andrews University. The subject seems to be popular, and it has attracted some excellent students. We cover the basics of deterministic discrete- and continuous-time linear and nonlinear models, both scalar equations and systems. Topics include analytic solutions of linear equations, equilibria, linearization, stability, phase portraits, bifurcations, simulations, and modeling methodology. We spend a good deal of time discussing the philosophy of science: how are mathematics and science different, how are they similar, and how should mathematics be used in science? We talk about logic, epistemology, and various notions of certainty. The students become familiar with the literature, work together in interdisciplinary research groups, and learn to give research talks. It would be nice to run a second semester of the course, coverfurcati311vthesluti21di even intellectually passionate, but they often feel insecure about the mathematics. And, it is pretty common for one or two lazy or anti-intellectual mathematics majors to enroll just because it sounds like an easy elective. These students sometimes attempt to cloak a refusal to learn with a mantle of mathematicians' disdain. (This is easy to qualitative analysis and phase space, bifurcations, and delay equations. Biological applications and classic topics include epidemiology, vaccination schemes, harvesting, delayed recruitment, Lotka-Voterra models, chemostats, competition, predator-prey systems, mutualism, Kolmogorov models, invasion and coexistence, the community matrix, and age structured McKendrick-Von Foerster models (including numerical schemes). Some chapters include case studies of such topics as the eutrophication of a lake, oscillations in flour beetle populations, Nicholson's blowflies, and the spruce budworm. Most chapters contain several interesting projects; for example, estimating the population of the U.S.A. and models for blood cell populations, neurons, and pulse vaccination.

The book reads as a well-written and fairly traditional undergraduate mathematics textbook, with theorems and some proofs (although many theorems are stated without proof). Its prerequisites are "a year of calculus, some background in elementary differential equations, and a little matrix theory." It would work well as a text for an upper division undergraduate topics course in applied dynamics, or as a graduate course for mathematically advanced ecology students. It served as an excellent reference and source for problems and projects in my own undergraduate interdisciplinary class. However, my students found it more difficult than Edelstein-Keshet [**6**].

The conclusion of the textbook hunt for my particular situation has been the following: (1) the kind of course I want to teach is too fluid to run in lockstep with a textbook; (2) no book will be at the right level for all the students in my class; indeed, there is no "right level"; (3) textbooks are useful for assigning readings and problems, as sources for student projects, and as reference books for the scholarly libraries of my upcoming young research biologists and applied mathematicians. In the Spring 2002 semester I used two texts: the book under review and Hastings [**9**]. I assigned readings and homework problems out of both books as appropriate, but did not base my lectures on either book. Instead, I ended up writing my own set of notes tailored to the interdisciplinary mix of students. This approach seemed to work well.

Brauer and Castillo-Chávez write in the preface: "This book is intended to inspire students in the biological sciences to incorporate mathematics in their approach to science .... A secondary goal is to expose students of mathematics to the process of modeling in the natural and social sciences." This statement cheers me, and I am reminded of the words of the evolutionary statistician R. A. Fisher [**7**, p. ix], when he said of mathematics and biology: "I can imagine no more beneficial change in scientific education than that which would allow each to appreciate something of the imaginative grandeur of the realms of thought explored by the other."

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