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Geometric Transient Solutions of Autonomous Scalar Maps

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K

1. THE PROBLEM

$$x(t+\square) = f(x(t)) \qquad (\square)$$

$$x(t) = \sum_{n=\square}^{\infty} c_n r^{nt}, \quad c_\square = x_e$$

†

[illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $< | \, | <$ [illegible]? [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible]?

[illegible] $+$ [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] $=$ [illegible] [illegible] [illegible]

[illegible]

$$x(t+\square) = f(a, x(t)) = ax(t) + h(a, x(t)) \qquad (\square)$$

[illegible]

$$h(a,x) = \sum_{i=\square}^{\infty} \frac{\square}{i!} \frac{\partial^i f}{\partial x^i}(a,\square)x^i.$$

[illegible]

[illegible] $\in$ [illegible] $\in$ [illegible] H [illegible]

2. LINEAR THEORY

[illegible] $\in$ [illegible]

$$x(t+\square) = ax(t) \qquad (\square)$$

$$x(t+\square) = ax(t) + b(t) \qquad (\square\square)$$

[illegible]

$$b(t) = \sum_{n=\square}^{\infty} d_n r^{nt}$$

[illegible] H [illegible]

[illegible] E [illegible] $\neq$ [illegible] $\in$ [illegible] H [illegible] H [illegible] H [illegible] H [illegible]

$$x(\square) = \sum_{n=\square}^{\infty} \frac{d_n}{r^n - a};$$

[redacted] $\in$ [illegible] H [illegible]

[illegible] $\neq$ [illegible] $\in$ [illegible] H [illegible]

$$\begin{aligned} x(t) &= a^t x(\square) + a^{t-\square} \sum_{n=\square}^{\infty} d_n \sum_{i=\square}^{t-\square} \left(\frac{r^n}{a}\right)^i \\ &= a^t x(\square) + a^{t-\square} \sum_{n=\square}^{\infty} d_n \frac{\square - (r^n/a)^t}{\square - (r^n/a)} \\ &= a^t \left(x(\square) - \sum_{n=\square}^{\infty} \frac{d_n}{r^n - a} \right) + \sum_{n=\square}^{\infty} \frac{d_n}{r^n - a} r^{nt} \end{aligned}$$

[illegible] H [illegible]

$$x(\square) = \sum_{n=\square}^{\infty} \frac{d_n}{r^n - a}$$

[illegible] $=$ [illegible] $\in$ [illegible] $=$ [illegible] H [illegible] $\in$ [illegible]

$$\begin{aligned} x(t) &= r^{mt} x(\square) + r^{m(t-\square)} \sum_{n=\square}^{\infty} d_n \sum_{i=\square}^{t-\square} \left(\frac{r^n}{r^m}\right)^i \\ &= r^{mt} x(\square) + r^{m(t-\square)} \sum_{\substack{n=\square \\ n\neq m}}^{\infty} d_n \frac{\square - (r^n/r^m)^t}{\square - (r^n/r^m)} + t d_m r^{m(t-\square)} \\ &= r^{mt} \left(x(\square) - \sum_{\substack{n=\square \\ n\neq m}}^{\infty} \frac{d_n}{r^n - r^m} \right) + \sum_{\substack{n=\square \\ n\neq m}}^{\infty} \frac{d_n}{r^n - r^m} r^{nt} + t d_m r^{m(t-\square)} \end{aligned}$$

[illegible] H [illegible] $=$ [illegible] □

[illegible] $H \longrightarrow H$ [illegible]

$$L\{x(t)\}_{t=\square}^{\infty} = \{x(t+\square) - a x(t)\}_{t=\square}^{\infty}$$

[illegible]

$$L\mathbf{x} = \mathbf{0} \qquad (\square)$$

$$L\mathbf{x} = \mathbf{b} \qquad (\square)$$

[illegible] $\mathbf{x}$, $\mathbf{b} \in H$ [illegible] $\mathbf{x} = \{x(t)\}_{t=\square}^{\infty}$ [illegible] $\mathbf{b} = \{b(t)\}_{t=\square}^{\infty}$

[illegible]

$$P\mathbf{b} = \mathbf{b} - \langle \mathbf{b}, \mathbf{v} \rangle \mathbf{v}$$

[illegible]

3. NONLINEAR THEORY

[illegible]

$$x(t + [illegible]) = f(a, x(t)) = ax(t) + h(a, x(t)) \quad ([illegible])$$

[illegible]

3.1. Bifurcations from Zero

[illegible]

$$x(t + [illegible]) - r^m x(t) = (a - r^m)x(t) + h(a, x(t)).$$

[illegible] $: H \longrightarrow H$ [illegible]

$$L\{x(t)\}_{t=□}^{\infty} = \{x(t+□) - r^m x(t)\}_{t=□}^{\infty}$$

[illegible] $B: H \longrightarrow H$ [illegible]

$$B(a, \{y(t)\}_{t=□}^{\infty}) = \{(a - r^m)y(t) + h(a, y(t))\}_{t=□}^{\infty}.$$

[illegible] $\mathbf{v} = \{r^{mt}\}_{t=□}^{\infty}$ [illegible]

[illegible]

$$L\mathbf{x} = B(a, \mathbf{x})$$

[illegible] $\mathbf{x} \in H$ [illegible] $=$ [illegible]

[illegible]

$$\mathbf{x} = \varepsilon\mathbf{v} + \varepsilon\mathbf{w}($$

[illegible]

$$\mathbf{0} = \mathbf{w}(\varepsilon) - L^{-1}PT(\varepsilon, \lambda, \mathbf{w})$$

$$\mathbf{0} = (I - P)T(\varepsilon, \lambda, \mathbf{w}).$$

[illegible] $\lambda(\varepsilon)$ [illegible] $\mathbf{w}(\varepsilon)$ [illegible] ε [illegible]

[illegible] Γ [illegible] $\times H \longrightarrow H$ [illegible]

$$\Gamma(\varepsilon, \lambda, \mathbf{w}) = (I - P)T(\varepsilon, \lambda, \mathbf{w}).$$

[illegible] Γ [illegible] $\mathbf{0}$ [illegible] Γ [illegible] λ [illegible] $\mathbf{0}$ [illegible]

$$\Delta\lambda \longmapsto \{r^{mt}\Delta\lambda\}_{t=1}^{\infty}$$

[illegible] $\lambda = \lambda(\varepsilon, \mathbf{w})$ [illegible] λ [illegible] $\mathbf{0}$ [illegible] $\lambda(\varepsilon, \mathbf{w})$ [illegible]

$$\mathbf{0} = \mathbf{w} - L^{-1}T(\varepsilon, \lambda(\varepsilon, \mathbf{w}), \mathbf{w}).$$

[illegible] $\mathbf{w} = \mathbf{w}(\varepsilon)$ [illegible] ε [illegible] $\mathbf{w}(\varepsilon) = \mathbf{0}$ [illegible]

[illegible]

[illegible] A [illegible] F [illegible] ε [illegible] $\times H$ [illegible] $= B$ [illegible] $\mathbf{0}$ [illegible] $\mathbf{0}$ [illegible]

[illegible] ε [illegible] $(a(\varepsilon), \mathbf{x}(\varepsilon)) = (a(\varepsilon), \{x(t,\varepsilon)\}_{t=1}^{\infty})$ [illegible] $\{x(t,\varepsilon)\}_{t=1}^{\infty} \in H_r$ [illegible] ε [illegible]

3.1.1. D[illegible] B[illegible]

[illegible] $\mathbf{0}$ [illegible] ε [illegible] ε [illegible] ε [illegible] $<$ [illegible]

[illegible]

$$w(t+\square,\varepsilon) - rw(t,\varepsilon) = \lambda(\varepsilon)[r^t + w(t,\varepsilon)] + \frac{\square}{\varepsilon} h(r+\lambda(\varepsilon), \varepsilon r^t + \varepsilon w(t,\varepsilon))$$

[illegible]

$$\square = \lambda(\varepsilon)$$

[illegible]

[illegible] $(a, \mathbf{x}(\varepsilon)) = (a, \{x(t, \varepsilon)\}_{t=[illegible]}^{\infty})$ [illegible]

[illegible]

3.2. Bifurcations from the Positive Equilibrium Branch

[illegible] $\mathbf{x} = B[illegible]\mathbf{x}$ [illegible]

$$y(t + [illegible]) = \alpha y(t) + \eta(y(t)) \qquad ([illegible])$$

[illegible] α [illegible] $\times H_{\alpha}$ [illegible] $\times H_{\alpha}$ [illegible] $\mathbf{x} = B[illegible]\mathbf{x}$ [illegible] $\mathbf{x}[illegible]\varepsilon[illegible] = \mathbf{y}[illegible]\varepsilon[illegible] + \mathbf{x}$ [illegible]

[illegible]

[illegible] $(a, \mathbf{x}(\varepsilon)) = (a, \{x(t, \varepsilon)\}_{t=[illegible]}^{\infty})$ [illegible]

3.3. *H*-stability

[illegible] ε[illegible]

[illegible] A [illegible] A [illegible] A [illegible] $+$ [illegible] $=$ [illegible] $\in$ [illegible] H [illegible] $\in$ [illegible] H [illegible] $=$ [illegible]

4. EXAMPLE: CALCULATING SOLUTIONS OF THE LOGISTIC MAP

[illegible]

$$x(t+\square) = ax(t)[\square - x(t)]$$

[illegible] $x(t) = \sum_{n=\square}^{\infty} c_n r^{nt}$ [illegible]

$$
\begin{aligned}
c_{\square} &= ac_{\square}(\square - c_{\square}) \\
rc_{\square} &= ac_{\square} - \square ac_{\square}c_{\square} \\
r^{\square}c_{\square} &= ac_{\square} - \square ac_{\square}c_{\square} - ac_{\square}^{\square} \\
&\vdots
\end{aligned}
$$

[illegible]

$$c_{\square} = \square \qquad c_{\square} = \square - \tfrac{\square}{a}$$

$$c_{\square} = \square \qquad r = a,\ c_{\square} = \varepsilon \qquad r = \square - a,\ c_{\square} = \varepsilon \qquad c_{\square} = \square$$

$$c_{\square} = \square \qquad r^{\square} = a,\ c_{\square} = \varepsilon \qquad c_{\square} = \frac{r}{r - r^{\square}}\varepsilon^{\square} \qquad c_{\square} = \frac{\square - r}{r - r^{\square}}\varepsilon^{\square} \qquad r^{\square} = \square - a,\ c_{\square} = \varepsilon \qquad c_{\square} = \square$$

[illegible] < [illegible] > [illegible]

[illegible]

[illegible] B [illegible]

[illegible] A I [illegible] D [illegible] 71 [illegible]

[illegible] J [illegible] D [illegible] E [illegible] A [illegible] 2 [illegible]

[illegible] J [illegible] I [illegible] E [illegible] 4 [illegible]

[illegible] I [illegible] A [illegible] D [illegible] C [illegible]